Angle Addition Identities

You can determine the value of a trigonometric function of a given angle when you can express the sum (or difference) of angles that can comprise that angle.

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}.$$

Source: ck12.org

For example, sin 75 = sin (30 + 45) = sin 30 cos 45 + sin 45 cos 30

$$= (1/2)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$=\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$=\frac{\sqrt{2}+\sqrt{6}}{4}$$

Double Angle and Half Angle Identities

Double angle identities are trig identities that can be used to rewrite trig functions that have a double angle.

Below are the double angle identities and an example of how they are used.

$$\sin(2\Theta) = 2\sin(\Theta)\cos(\Theta)$$

$$\cos(2\Theta) = \cos^2(\Theta) - \sin^2(\Theta)$$

$$= 1 - 2\sin^2(\Theta)$$

$$= 2\cos^2(\Theta) - 1$$

$$\tan(2\Theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Example: Given $\sin(\Theta) = \frac{4}{5}$ and $0 < \Theta < \frac{\pi}{2}$ find $\sin(2\Theta)$

$$\sin(2\Theta) = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= \frac{24}{25}$$

The half-angle formulas can be used in the same way.

Half-Angle Formulas

$$\sin(\theta/2) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos(\theta/2) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan(\theta/2) = \pm\frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

Source: ck12.org