Conditional Probability: Given That statements - P(F|E) means to find the probability that event F occurs within event E (or we can say probability event F occurs but is **restricted** to the outcomes that occur **only** in event E)

Formula $P(F|E) = \dfrac{P(E \text{ and } F)}{P(E)}$ or

Option 2 – P(E|F) [**is not the same as P(F|E)**] $= \dfrac{P(E \text{ and } F)}{P(F)}$ * This formula is restricted to outcomes in **only** event F

****Tip to help set up when using Formula** $\mathbf{P(F|E) = \dfrac{P(E \text{ and } F)}{P(E)}}$ - Look at the phrase itself and if the word "given" is used, then whatever event comes ***before*** the word "GIVEN" is event E and the event that comes ***after*** the word "GIVEN" is event F

Ex: Josh is playing backgammon, a game played with 2 six-sided dice. What is the probability that the sum of the 2 dice he rolls is less than 4 GIVEN that he rolls an odd number?

Event F **Event E (Total)(Denom)**

F: outcomes of sum being less than 4 are: (1,2)(2,1)(1,1)

E: outcomes of sum being odd:
(1,2)(2,1)(1,4)(4,1)(1,6)(6,1)(2,3)(3,2)(2,5)(5,2)(3,4)(4,3)(3,6)(6,3)(4,5)(5,4)(5,6)(6,5) = 18 **total** outcomes

2 outcomes occur in both, so our Probability is $\frac{2}{18} = \frac{1}{9}$ $or\ .111$

Using formula $P(F|E) = \dfrac{P(E \text{ and } F)}{P(E)}$ $\dfrac{\frac{2\ (both\ less\ than\ 4\ \&\ odd)}{12\ (total\ for\ both\ dice)}}{\frac{18\ \ (total\ of\ odd\ sums)}{12\ (total\ for\ both\ dice)}} = \frac{2}{12}\ x\ \frac{12}{18} = \frac{2}{18}\ or\ .111$

Ex. **2**: A box of markers contains 10 black-inked (4 wide-tipped & 6 fine-tipped) and 15 red-inked (3 wide-tipped & 12 fine-tipped). What is the probability that a randomly chosen marker will be red GIVEN that it is fine tipped?

Event F **Event E (Total)(Denom)**

F: outcome for red-inked markers is 15 (3 wide-tipped & 12 fine-tipped)

E: outcome for fine-tipped markers is 18 **total** (6 black & 12 red)

2 outcomes occur in both, so our probability is $\frac{12}{18} = \frac{2}{3}$ $or\ .667$

Using formula P(F|E) = $\frac{P(E \text{ and } F)}{P(E)}$ $\frac{\frac{12\ (both\ black\ \&\ red)}{25\ (total\ black\ \&\ red)}}{\frac{18\ (total\ fine\text{-}tipped)}{25\ \ (total\ black\ \&\ red)}} = \frac{12}{25}\ x\ \frac{25}{18} = \frac{12}{18}\ or\ .667$

*Phrases **<u>without</u>** the word(s) "given" or "given that" use Formula P(E|F) = $\frac{P(E \text{ and } F)}{P(F)}$

Ex. In a sample of 40 vehicles, 18 are red, 6 are trucks, and 2 are both. Suppose that a randomly <u>selected vehicle is red</u>. What is the probability it is a truck?

Event F (Total) (Denom)

Event E

F: outcome of red vehicles is 18

E: outcome of trucks is 6

2 outcomes occur in both, so our probability is $\frac{2}{18} = \frac{1}{9}\ or\ .111$ ** this time we are using event F as our total because we are restricted to only vehicles that are red per the statement.

Using formula P(E|F) = $\frac{P(E \text{ and } F)}{P(F)}$ $\frac{\frac{2\ (both\ are\ red\ \&\ a\ truck)}{40\ (total\ of\ all\ vehicles)}}{\frac{18\ \ (total\ red\ vehicles)}{40\ (total\ of\ all\ vehicles)}} = \frac{2}{40}\ x\ \frac{40}{18} = \frac{2}{18}\ or\ .111$

Ex. 2: 70% of your friends like Chocolate, and 35% like Chocolate AND like Strawberry.

Event F (Total) (Denom)

Event F & E

What percent of <u>those who like Chocolate</u> also like Strawberry?

F: outcome like Chocolate

E: Did not indicate outcome in statement

2 outcomes occur in both, so our probability is $\frac{.35}{.70} = .5\ or\ 50\%$ ** again, we are using event F as our total because we are restricted to only those who like chocolate, per the statement.

Using formula P(E|F) = $\frac{P(E \text{ and } F)}{P(F)}$ $\frac{.35\ (like\ both\ choc.\&\ straw.)}{.70\ (like\ chocolate)} = .5\ or\ 50\%$

Given That vs. AND problems

	Speeding Ticket	No Speeding Ticket	Total
Blue Car	120	129	249
Not a Blue Car	184	163	347
Total	304	292	596

Find the probability that a randomly chosen person…

1. Has a blue car given that they got a speeding ticket.

Event F **Event E**

Formula $P(F|E) = \frac{P(E \text{ and } F)}{P(E)} = \frac{120\ (blue\ car\ and\ \ speeding\ ticket)}{304\ (total\ that\ got\ a\ speeding\ ticket} = .3947\ or\ 39.5\%$

**remember GIVEN THAT statements are RESTRICTED to the outcomes that happen in event E

2. Has a blue car AND got a speeding ticket.

The first part of the question above asks about the person, so our TOTAL (denominator) is total amount of people...596

Then the second part of the question asks, has a blue car AND got a speeding ticket, so the number that fits both criteria is…120

$$\frac{120\ (has\ a\ blue\ car\ and\ got\ a\ speeding\ ticket)}{596\ (total\ amount\ of\ people)} = .2013 \text{ or } 20.1\%$$

Hope this helps! Let us know at general_tutoring@eastcentral.edu