Problems # 1-71 and #115-119 are a review for the concepts covered on the midterm

# PRE-CALCULUS FINAL REVIEW – ALGEBRA PORTION

**Find (a) the distance between points P and Q, and (b) determine the midpoint M of the segment joining P and Q.**

1. $P(-6,-10),\ Q(6,5)$
2. $P(12y,-3y),\ Q(20y,12y),\quad y>0$

**Find the equation of the line (in slope-intercept form) satisfying the given conditions.**

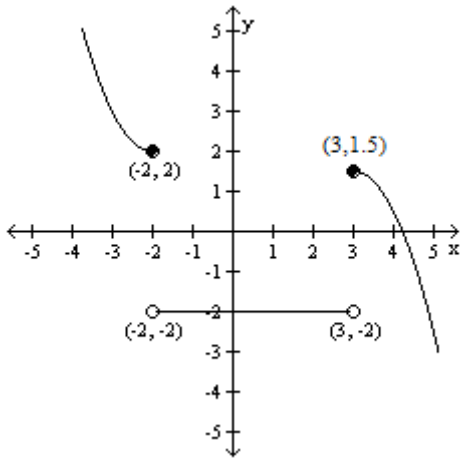
3. Passing through $\left(\frac{3}{4},\frac{1}{4}\right)$ and perpendicular to the line passing through $(-3,-5)$ and $(-4,0)$.
4. Passing through $(5,-4)$ and parallel to $3x-5y+7=0$.

**Solve**

5. $-4(3x+9) \geq -3(4x+3)$
6. $-4(2x+8) \leq -2(4x+4)$

**Use the graph to answer the following questions**

7. Determine the intervals on which the following function is (a) increasing, (b) decreasing, and (c) constant. Then give the (d) domain and (e) range.




**Determine if the following functions are even, odd, or neither.**

8. $f(x)=3x^5-x^3+7x$
9. $f(x)=\sqrt{x^2+1}$
10. $f(x)=x^4-5x+2$
11. $f(x)=|5x|$
12. $f(x)=\frac{x^5}{x^4+7}$

**Give the equation of the function whose graph is described.**

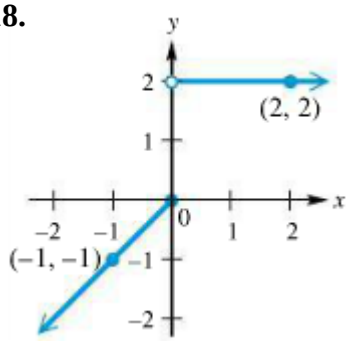
13. The graph of $y = \sqrt[3]{x}$ is shifted 2 units to the left. This graph is then vertically stretched by applying a factor of 1.5. Finally, the graph is shifted 8 units upward.

**Solve.**

14. $|3(x-5)+2|+3=9$
15. $\left|\frac{3}{4}x-\frac{1}{2}\right|<-4$
16. $3|2x+5|-8>-5$
17. $\left|x-\frac{1}{2}\right|=\left|\frac{1}{2}x-2\right|$

**Write a piecewise-defined function that represents the graph below. Then give the domain and range**

**18.**




**Given $f(x)=x^3$ and $g(x)=2x-1$ find the following:**

19. $(f-g)(2)$
20. $(fg)(-1)$
21. $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$
22. $(f\circ g)(-2)$

**Given $\mathbf{f(x)=\frac{2}{x-1}}$ and $\mathbf{g(x)=\frac{3}{x}}$ find the following:**

23. $(f°g)(x)$ and the domain of $f°g$

**Find the average rate of change for the given function over the given interval**

24. $f(x)=x^2-1$ from $x=-2$ to $x=3$
25. $f(x)=x^2-1$ from $x=-2$ to $x=-2+h$

**Find the difference quotient for the following function**

26. $f(x) = 2x^2 - 3x$

**Simplify. Write each expression in the form** $a+bi$.

27. $-5i(4-3i)^2$
28. $(3-i)(3+i)(2-6i)$
29. $\dfrac{-19-9i}{i}$
30. $\dfrac{4-3i}{4+3i}$
31. $\dfrac{\sqrt{-10}\sqrt{-5}}{\sqrt{2}}$

**Find the equation of the quadratic function satisfying the given conditions. Write your answer in the form** $P(x) = ax^2 + bx + c$.

32. Vertex: $(-6,-12)$; through $(6,24)$

**Write the following function in vertex form**

33. $f(x) = 3x^2 + 3x - 6$

**Find the vertex of the graph of the following function**

34. $f(x) = 2x^2 - 4x + 5$

**Solve.**

35. $(4x+1)^2 = 20$
36. $(x+4)(x-1) = -5x-4$
37. $x^2 - 2x = -5$
38. $s = s_0 + gt^2 + k$ for $t$
39. $x^2 + 4x \le -3$
40. $x^2 - x + 1 < 0$

**Use the function** $f(x) = -2x^4 + 2x^3$ **to answer the following.**

41. Use the leading coefficient test to determine the graph's end behavior.
42. Find the x-intercept(s).
43. Find the y-intercept(s).
44. Determine whether the graph has y-axis symmetry, origin symmetry, or neither.
45. What are the maximum number of turning points?

**Divide.**

46. $\frac{-x^3-x-5}{x+1}$

47. $\frac{x^7+1}{x+1}$

48. $\frac{3x^4-2x^2-5}{3x^2-5}$

**Use synthetic division to find P(k)**

49. $P(x) = 5x^3 + 2x^2 - x + 5,\ k = -2$

**Find all zeroes of the following function**

50. $P(x) = 3x^3 + 5x^2 - 3x - 2$ given that $-2$ is a zero

**Completely factor the following function**

51. $f(x) = 24x^3 + 40x^2 - 2x - 12$

**Find a cubic polynomial with the following zeroes:**

52. 4 and $2 + i$

**Sketch a rough graph of the following function by finding the end behavior, zeroes, and their multiplicity**

53. $f(x) = x^3 + x^2 - 8x - 12$

**Solve each equation and inequality. Assume that *k* is a positive constant.**

54. $2x^3 = 4x^2 - 2x$

55. $x^3 - 3x^2 - 6x + 8 = 0$

56. $x^3 - 3x^2 - 6x + 8 > 0$

**Find all asymptotes (horizontal, vertical, and slant) of the following functions.**

57. $g(x) = \frac{5x^2+4x+3}{5x+9}$

58. $f(x) = \frac{x-2x^2-11}{x^2-3}$

**Solve.**

59. $\frac{1}{x+2} + \frac{3}{x+7} = \frac{5}{x^2+9x=14}$

60. $\frac{2x-5}{x^2-1} \geq 0$

61. $6 - \frac{2}{x+3} = \frac{3x-5}{x+3}$

62. $\frac{1}{x-1}+\frac{1}{x+1}>\frac{3}{4}$

63. $\frac{3}{x+1}<\frac{4}{x+2}$

**Determine the domain of each function.**

64. $f(x)=-\sqrt[4]{2-0.5x}$

65. $f(x)=\sqrt[5]{x+32}$

66. $f(x)=\sqrt{81-x^2}$

**Simplify the following expression so that it does not contain negative exponents**

67. $(x+3)^{-\frac{1}{5}}+(x+3)^{-\frac{2}{5}}$

**Solve.**

68. $\sqrt{x+5}+1=x$

69. $\sqrt{2x+3}-\sqrt{x+1}=1$

70. $x^{3/4}-2x^{1/2}-4x^{1/4}+8=0$

71. $3x^{-2}-19x^{-1}+20=0$

**Determine if the following functions are one-to-one. If so, find their inverse. If not, state "not one-to-one"**

72. $y=-x^2+2$

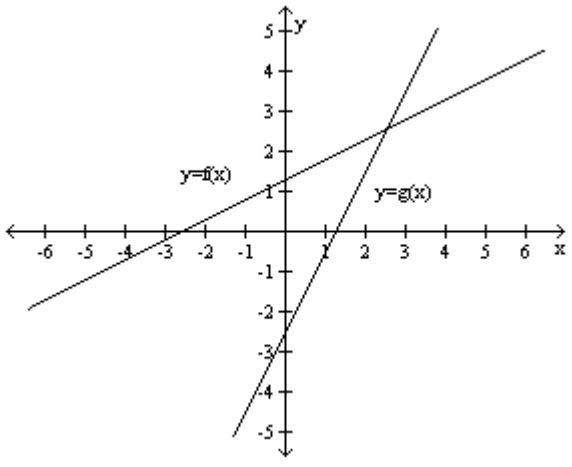
73. $f(x)=-\sqrt{x^2-16},\ x\geq 4$

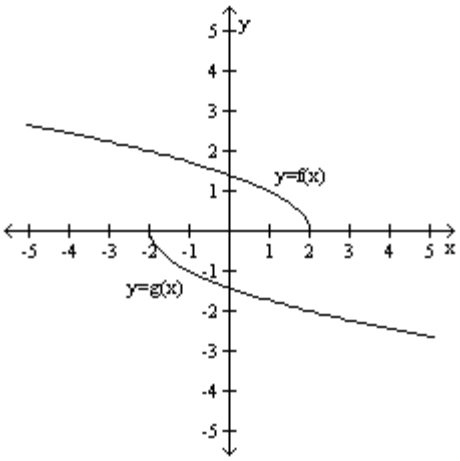
74. $f(x)=\frac{4-x}{5x}$

75. $f(x)=\frac{2x+1}{x-1}$

**Decide whether the pairs of functions are inverses.**

76.



77. 


**Use the properties of logarithms to expand the expression completely, if possible**

78. $\log_2 \frac{6x}{y}$
79. $\log_6(7m+3q)$
80. $\log_2 \frac{2\sqrt{3}}{5p}$
81. $\log_p \sqrt[3]{\frac{m^5}{kt^2}}$

**Use the properties of logarithms to rewrite each expression as a single logarithm with coefficient 1.**

82. $\log_b(2y+5)-\frac{1}{2}\log_b(y+3)$
83. $-\frac{2}{3}\log_5 5m^2+\frac{1}{2}\log_5 25m^2$
84. $\frac{4}{3}\ln m-\frac{2}{3}\ln 8n-\ln m^3n^2$

**Find the domain of each logarithmic function.**

85. $y=\ln(x^4+8)$
86. $y=\log(x^3-81x)$
87. $y=\log\left(\frac{x+1}{x-5}\right)$
88. $y=\log|6x+6|$

**Use the change-of-base rule to find an approximation for each logarithm to 3 decimal places.**

89. $\log_{200} 175$
90. $\log_{5.8} 12.7$

**Solve. Express all solutions in exact form.**

91. $5^{2x+1} = 25$
92. $e^{5x-7} = (e^3)^x$
93. $\left(\frac{1}{2}\right)^x = 5$
94. $3^{x-4} = 7^{2x+5}$
95. $\left(\frac{1}{3}\right)^x = -3$
96. $5(1.2)^{3x-2} + 1 = 11$
97. $3^{2x} + 35 = 12(3^x)$
98. $(\log_2 x)^2 + \log_2 x = 2$
99. $\log_5(8-3x) = 3$
100. $\ln x + \ln x^2 = 3$
101. $\ln(4x - 2) - \ln 4 = -\ln(x - 2)$
102. $2\log_2(5x-3) + 1 = 17$
103. $\frac{1}{4}e^{2x} + 2e^x = 3$
104. $\log_5(x + 2) + \log_5(x - 2) = 1$

**Solve each formula for the indicated variable.**

105. $T = T_0 + (T_1 - T_0)10^{-kt}$, for $t$
106. $y = \frac{K}{1 + ae^{-bx}}$, for $b$
107. $d = 10\log\left(\frac{I}{I_0}\right)$, for $I$

**Sketch a graph of the following function**

108. $f(x) = \left(\frac{1}{3}\right)^{x+2}$

**Solve each system.**

109. $\begin{aligned} 4x-5y&=-11 \\ 2x+\ y&=5 \end{aligned}$

110. $\begin{aligned} 2x\ -7y\ &=8 \\ -3x+\frac{21}{2}y&=5 \end{aligned}$

111. $\begin{aligned} 4x-\ y+3z&=-2 \\ 3x+5y-\ z&=15 \\ -2x+\ y+4z&=14 \end{aligned}$

**Determine whether each equation has a circle as its graph. If it does, give the center and radius.**

112. $9x^2+12x+9y^2-18y-23=0$

113. $x^2+4x+y^2-8y+32=0$

114. $4x^2+4x+4y^2-4y-3=0$

**Applications.**

115. The manager at a restaurant earns 15% more than the chef. Together, their salaries are $78,475. How much is the manager's salary?

116. How much water should be added to 8 milliliters of 6% saline solution to reduce the concentration to 4% saline?

117. A baseball is hit so that its height in feet after $t$ seconds is $s(t) = -16t^2 + 44t + 4$. How high is the baseball after 1 second? Find the maximum height of the baseball.

118. A raised wooden walkway is being constructed through a wetland. The walkway will have the shape of a right triangle with one leg 700 yards longer than the other and the hypotenuse 100 yards longer than the longer leg. Find the total length of the walkway.

119. You have 80 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

120. Determine which of the following two plans will provide a better yield.

(a) Plan A: $40,000 invested for 3 years at 2.5%, compounded quarterly
Plan B: $40,000 invested for 3 years at 2.4%, compounded continuously

(b) Plan A: $50,000 invested for 10 years at 4.75%, compounded daily $(n=365)$
Plan B: $50,000 invested for 10 years at 4.7%, compounded continuously

121. Find the present value of an account that will be worth $\$25,000$ in 2.75 years, if interest is compounded quarterly at 6%.

122. How long will it take for the amount in an investment to double if the investment earns 2.5% compounded quarterly?

123. The growth of bacteria in food products makes it necessary to date some products (such as milk) so that they will be sold and consumed before the bacterial count becomes too high. Suppose that, under certain storage conditions, the number of bacteria present in a product is $f(t) = 500e^{0.1t}$, where $t$ is time in days after packing of the product and the value of $f(t)$ is in millions. If the product cannot be safely eaten after the bacterial count reaches 3,000,000,000, how long will this take?

124. A student invests a total of $5,000 at 3% and 4% annually. After 1 year, the student receives a total of $187.50 in interest. How much did the student invest at each interest rate?

**Answer Key**

1. (a) $3\sqrt{41}$, (b) $\left(0,-\frac{5}{2}\right)$
2. (a) 17y, (b) $\left(16y,\frac{9}{2}y\right)$
3. $y=\frac{1}{5}x+\frac{1}{10}$
4. $y=\frac{3}{5}x-7$
5. $\emptyset$
6. $(-\infty,\infty)$
7. (a) none (b) $(-\infty,-2);\ (3,\infty)$ (c) $(-2,3)$ (d) $(-\infty,\infty)$ (e) $(-\infty,1.5]\cup[2,\infty)$
8. Odd
9. Even
10. Neither
11. Even
12. Odd
13. $y=1.5\sqrt[3]{x+2}+8$
14. $\left\{\frac{7}{3},\frac{19}{3}\right\}$
15. No solution
16. $(-\infty,-3)\cup(-2,\infty)$
17. $\left\{-3,\frac{5}{3}\right\}$
18. $f(x)=\begin{cases}x \text{ if } x\le 0\\ 2 \text{ if } x>0\end{cases}$ Domain: $(-\infty,\infty)$ Range: $(-\infty,0]\cup\{2\}$
19. 5
20. 3
21. Undefined
22. $-125$
23. $(f\circ g)(x)=\frac{2x}{3-x}$ Domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$
24. 1
25. $h-4$
26. $4x+2h-3$
27. $-120-35i$
28. $20-60i$
29. $-9+19i$
30. $\frac{7}{25}-\frac{24}{25}i$
31. $-5$

32. $P(x)=\frac{1}{4}x^2+3x-3$

33. $f(x)=3\left(x+\frac{1}{2}\right)^2-\frac{27}{4}$

34. $(1,3)$

35. $\left\{\frac{-1\pm2\sqrt{5}}{4}\right\}$

36. $\{-8,0\}$

37. $\{1\pm2i\}$

38. $t=\frac{\pm\sqrt{(s-s_0-k)g}}{g}$

39. $[-3,-1]$

40. $\emptyset$

41. As $x$ approaches $\pm\infty$, $f(x)$ approaches $-\infty$

42. $(0,0)$ and $(1,0)$

43. $(0,0)$

44. Neither

45. 3

46. $-3x^2+3x-4+\frac{-1}{x+1}$

47. $x^6-x^5+x^4-x^3+x^2-x+1$

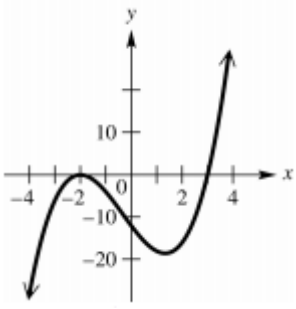
48. $x^2+1$

49. $-25$

50. $x=-2,\frac{1\pm\sqrt{13}}{6}$

51. $f(x)=2(2x+3)(3x+2)(2x-1)$

52. $P(x)=x^3-8x^2+21x-20$

53. 


54. $x=0,1$

55. $x=-2,1,4$

56. $(-2,1)\cup(4,\infty)$

57. VA: $x=-\frac{9}{5}$ HA: None SA: $y=x-1$

58. VA: $x=\sqrt{3},\ x=-\sqrt{3}$ HA: $y=-2$ SA: None

59. $\emptyset$

60. $(-1,1)\cup\left[\frac{5}{2},\infty\right)$

61. $x=-7$

62. $\left(-1,-\frac{1}{3}\right)\cup(1,3)$

63. $(-2,-1)\cup(2,\infty)$

64. $(-\infty,4]$

65. $(-\infty,\infty)$

66. $[-9,9]$

67. $\frac{(x+3)^{\frac{1}{5}}+1}{(x+3)^{\frac{2}{5}}}$

68. $x=4$

69. $x=-1,3$

70. $\{16\}$

71. $\left\{\frac{1}{5},\frac{3}{4}\right\}$

72. Not one-to-one

73. $f^{-1}(x)=\sqrt{x^2+16},\ x\le 0$

74. $f^{-1}(x)=\frac{4}{5x+1}$

75. $f^{-1}(x)=\frac{x+1}{x-2}$

76. Yes

77. No

78. $\log_2 6+\log_2 x-\log_2 y$

79. Cannot be rewritten

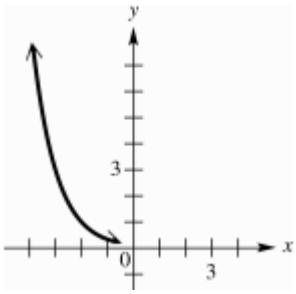
80. $1+\frac{1}{2}\log_2 3-\log_2 5-\log_2 p$

81. $\frac{1}{3}(5\log_p m-\log_p k-2\log_p t)$

82. $\log_b\frac{2y+5}{\sqrt{y+3}}$

83. $\log_5(5^{1/3}m^{-1/3})$, or $\log_5\sqrt[3]{\frac{5}{m}}$

84. $\ln\sqrt[3]{\frac{1}{64m^5n^8}}$

85. $(-\infty, \infty)$
86. $(-9, 0) \cup (9, \infty)$
87. $(-\infty, -1) \cup (5, \infty)$
88. $(-\infty, -1) \cup (-1, \infty)$
89. 0.975
90. 1.446
91. $x = \frac{1}{2}$
92. $x = \frac{7}{2}$
93. $x = \frac{\log 5}{\log\left(\frac{1}{2}\right)}$
94. $\left\{\frac{5\log 7 + 4\log 3}{\log 3 - 2\log 7}\right\}$
95. No solution
96. $\frac{2}{3} + \frac{\log 2}{3\log 1.2}$
97. $\log_3 5, \log_3 7$
98. $x = \frac{1}{4}, 2$
99. $\{-39\}$
100. 3
101. 2.5
102. $\left\{\frac{259}{5}\right\}$
103. $\left\{\ln\left(2\sqrt{7} - 4\right)\right\}$
104. 3
105. $t = -\frac{1}{k}\log\left(\frac{T - T_0}{T_1 - T_0}\right)$
106. $b = \frac{\ln\left(\frac{K - y}{ay}\right)}{-x}$
107. $I = I_0 \cdot 10^{d/10}$
108. 

109. $\{(1,3)\}$
110. No solution
111. $\{(-1,4,2)\}$
112. Yes; $\left(-\frac{2}{3},1\right)$; 2
113. No
114. Yes; $\left(-\frac{1}{2},\frac{1}{2}\right)$; $\frac{\sqrt{5}}{2}$
115. \$41,975
116. 4 mL
117. 32 ft, 34.25 ft
118. 3000 yards
119. 20 yards by 20 yards; 400 yards squared
120. (a) Plan A is better by \$119.09.
    (b) Plan A is better by \$398.52.
121. \$21,223.33
122. 27.81 years
123. 18 days
124. \$1250 at 3%; \$3750 at 4%